Determining the resistivity of a wire

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I. INTRODUCTION

In electrical circuits, the resistance of any piece of wire is often assumed to be zero. While this is a good approximation, as the resistance values for other parts of the circuit dwarf the resistance of the wire, the wire does in fact have some resistance. With sufficiently sensitive equipment, this resistance can be measured, or calculated from other measured values. This resistance can then be used to determine the resistivity, an intrinsic physical property, of the material of which the wire is composed.

One way of determining the resistance of a piece of wire is to connect it in series with another resistor and measure the voltage across the wire. While in theory this could be done with a sufficiently sensitive digital multimeter, this method fails to account for environmental effects (e.g. noise induced by unshielded electrical wiring) that could potentially skew the results. Using a function generator, however, to generate an AC signal, and a lock-in amplifier to filter ambient noise, this voltage can be measured much more accurately. The resistivity of a substance is related to the resistance of a sample of that substance by[1]

$$\rho = \frac{RA}{L} \tag{1}$$

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where ρ is the resistivity of the substance, R is the measured resistance of the substance, A is the cross-sectional area of the sample, and L is the length of the sample. As the sample is a length of wire with circular cross-section, we can substitute the formula for the area of a circle for A, giving

$$\rho = \frac{\pi R d^2}{4L} \tag{2}$$

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II. PROCEDURE

On a breadboard, we set up a circuit with a 1.02 kΩ resistor connected in series with our wire sample. We powered the circuit with a function generator[2], set to generate sinusoidal AC signal. We connected the reference output of the function generator to the reference input of a lock-in amplifier, and connected the lock-in amplifier across the wire sample. We used two samples of coated copper magnet wire, of different lengths and thicknesses, wound aninductively. Using a digital multimeter, we measured the voltage across the 1.02 kΩ resistor. We used the lock-in amplifier to measure the voltage across the wire sample. We took five trials for each sample.

TABLE I: Physical characteristics of samples

Sample	diameter (mm)[a]	length (cm)
Bright	0.595	297.6
Dark	0.515	142.1

[a]For each sample, the outer coating was burned off of a section of the sample. The burned section was cleaned, and its diameter measured using a micrometer.

III. CALCULATIONS

As we chose to measure the voltage drop across the 1.02 $k\Omega$ resistor, rather than directly measuring the current through the circuit, we calculated the current through the circuit using Ohm's Law, $I = \frac{V_R}{R}$, where I is the current, V_R is the voltage across the resistor, and R is the resistance of said resistor, and used this current to calculate the resistance of the wire sample,

$$R_W = \frac{V_W R}{V_R} \tag{3}$$

where R_W is the resistance of the wire, V_W is the voltage measured across the wire by the lock-in amplifier, R is the resistance of the resistor (1.02 $k\Omega$), and V_R is the voltage measured across the resistor by the digital multimeter. This, along with the formula for the area of a circle as mentioned previously, can then be substituted into (2), giving

$$\rho = \frac{\pi V_W R d^2}{4 V_R L} \tag{4}$$

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[2] Lockin Amplifier Model SR510, manufactured by Stanford Research Systems of Sunnyvale, CA

where ρ is the resistivity of the substance composing the wire sample, V_W is the voltage measured across the wire by the lock-in amplifier, R is the resistance of the resistor (1.02 $k\Omega$), d is the diameter of the wire (assumed to be constant throughout the sample), V_R is the voltage measured across the resistor by the digital multimeter, and L is the length of the wire sample. We calculated the uncertainty in ρ by

$$\Delta\rho = \sqrt{\left(\frac{\delta\rho}{\delta V_W}\Delta V_W\right)^2 + \left(\frac{\delta\rho}{\delta R}\Delta R\right)^2 + \left(\frac{\delta\rho}{\delta d}\Delta d\right)^2 + \left(\frac{\delta\rho}{\delta V_R}\Delta V_R\right)^2 + \left(\frac{\delta\rho}{\delta L}\Delta L\right)^2} \quad (5)$$

TABLE II: data and calculated values for ρ

Sample	f (Hz)	V_W (μV)	V_R	$\rho \times 10^8$	$\Delta\rho(\%)$	Phase angle
Dark	523	41.4	398.47	1.54716	1.02355	7.4°
Dark[a]	523	40.6	399.47	1.51347	1.03559	7.8°
Dark	323	40.9	399.14	1.52591	1.03557	9.1°
Dark	723	41.5	399.19	1.54810	1.03558	9.0°
Dark	1323	44.0	397.49	1.64838	1.03567	−1.4°
Bright	523	60.8	399.47	1.444455	1.02355	9.6°
Bright	323	60.4	399.77	1.43397	1.02354	9.6°
Bright	723	61.9	399.24	1.47153	1.02356	10.0°
Bright[b]	323	61.0	399.93	1.44763	1.02354	10.3°
Bright	1323	62.6	397.39	1.49510	1.02367	−1.2°

[a]the ends of both samples were resanded with 100 grit Al_2O_3 sandpaper after the first measurement of the dark sample. Thus the dark sample was measured twice at 523 Hz, once before sanding and once afterward.

[b]We decided to perform additional trials after our samples had been unwound in order to measure their length. The samples were aninductively rewound, and additional measurements were taken.

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IV. RESULTS AND POTENTIAL SOURCES OF ERROR

The results of each individual trial are detailed in the table below.

For our "bright" sample, we calculated a mean value for ρ of $(1.459 \pm 0.015) \times 10^{-8} \Omega m$. For our "dark" sample, we calculated a mean value for ρ of $(1.556 \pm 0.016) \times 10^{-8} \Omega m$. Both samples were copper magnetic wire with nominal resistivity $1.678 \times 10^{-8} \Omega m$. This is a difference of 7.27% in the case of the dark sample, and a difference of 13.1% in the bright sample.

One potential source of error is impurities in the copper wire. It might be expected that impurities would tend to increase the resistance of the wire, thus increasing the wire's resistivity, but we feel it equally likely that impurities would decrease the resistivity as well. However this fails to explain how, if both samples are of the same material, the calculated values for ρ differ by over 10%. While both samples are of copper magnetic wire, each sample comes from a different spool. Thus it is possible that each sample would contain different impurities.

Another potential source of error, systematic this time, lies in our procedure. While we took pains to adjust the phase angle on the lock-in amplifier to minimize any external (e.g. inductive effects from ambient electromagnetic fields) effects, we did not adjust the frequency on the function generator in order to minimize this phase angle. It is worth noting, however, that our maximum phase angle was only 10.3°, in the case of the bright sample, and 9.1° in the case of the dark, and that the standard deviation of our calculated values for ρ is only 1.7% of the mean, in the case of the bright sample, and 3.9% in the case of the dark.

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[1] Nave, C. Rod. "Resistance and Resistivity." From HyperPhysics. http://hyperphysics.phy-astr.gsu.edu/hbase/electric/resis.html#c2 [Accessed 1 April 2005]

[2] 21.5 MHz Multi-Function Arbitrary Waveform Generator Model 4070A, manufactured by BK Precision Corporation of Yorba Linda, CA