# Determining the fundamental electric charge

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(Dated: May 2, 2005)

By measuring the terminal velocity of droplets of mineral oil sprayed into a hollow chamber, and subsequently measuring the upward velocity of those droplets when an electric field of known magnitude was applied, we were able to determine the charge on each droplet. We used an ionizing source to change the charge on some of the droplets, then measured their charge again. We did this a number of times for each droplet. Visual inspection of our data showed evidence of quantization, so we grouped the individual charge values into eight bins of varying sizes. The mean difference between the mean values of each bin was $(1.72 \pm 0.19) \times 10^{-19}$ C. This is in agreement with the accepted value for the fundamental electric charge, $1.602 \times 10^{-19}$ C.

## I. INTRODUCTION

In 1897, J.J. Thomson discovered the electron. He was able to experimentally determine the ratio of its charge to its mass, but neither were known separately. In 1909, Robert Millikan was able to determine the charge on the electron.

Millikan's method was to spray tiny oil droplets into the air above two charged plates separated by an insulating ring. The top plate had a small hole in it, and it was through this hole that a few droplets of oil would go through. By shining a bright light source into the chamber, the oil drops were illuminated, and could be observed through a microscope. Millikan first observed these droplets with no electric field present. The droplets quickly reached terminal velocity and this could be used to calculate the droplets' mass. When the electric field was then turned on, Millikan could vary the strength of the field until the drops were stationary. This allowed him to determine the charge on the drops.

Our apparatus[1] is similar to that of Millikan. It has two metal plates separated by a clear plastic spacer. This spacer is hollow in the center, creating a chamber that oil droplets can enter through a small hole in the top plate. The entire chamber is protected by a removable cylindrical shield that contains two circular windows. When the shield is properly aligned, one of the windows allows a light, attached to the apparatus near the chamber, to shine into the chamber and illuminate its contents. This window is dichroic in order to minimize heating of the chamber. The chamber is viewed with an attached microscope. There are two parallel lines etched on the reticle of this miscroscope. On the side of the chamber opposite the light is a switch allowing one to allow radiation from the built-in ionizing source to enter the chamber.

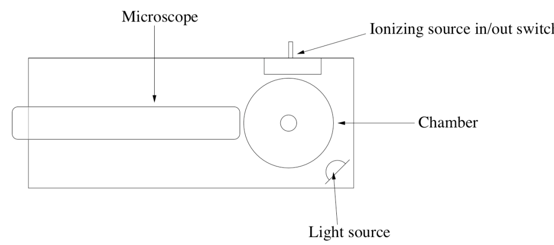


FIG. 1: Top view of apparatus

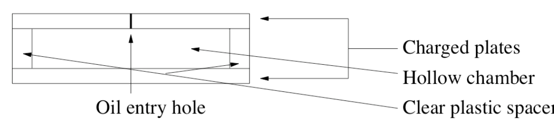


FIG. 2: Closeup view of chamber

## II. PROCEDURE

We began by determining values for $\rho$, $\eta$, $P$, and $d$, all of which remained constant througout the experiment. We found $\rho = 870 \frac{kg}{m}$. We found $\eta = 1.8365 \times 10^{-5} \frac{N \cdot s}{m^2}$[2] We found $P = 1.016 \times 10^5 Pa$[3]. We found $d = 5.60 \times 10^{-3} m$.

After the constant values had been determined, we began to spray drops into the chamber. We placed the provided atomizer, which was filled with mineral oil[4], above the entry hole for the chamber and sprayed until we were able to see a droplet through the microscope. We determined the droplet's terminal velocity by applying an electric field to the drop sufficient in intensity to cause it to rise. When the drop had risen above the top line marked on the reticle of the microscope, we removed the electric field and allowed the drop to fall until it had passed below the bottom line. We timed this fall. We then applied the electric field again, causing the drop to rise at constant velocity. We timed this rise, then removed the electric field and allowed the drop to fall once again. After we had repeated the sequence of causing the drop to rise and allowing it to fall a number of times, we then allowed ionizing radiation to enter the chamber for

*URL: `http://www.cs.cofc.edu/~cdpeters` a brief period of time. This caused the air to become ionized, and the charge on the oil drop to change. We caused the drop to rise and fall again, timing both. We determined the charge on the droplet after each sequence of rising and falling. Note that while we theoretically could have changed the charge on the droplet inifinitely many times, in practice we often lost sight of the drop not long after the second or third time the charge was changed.

## III. DERIVATION OF Q

Stokes' Law deals with the force due to friction for very small particles in a viscous fluid, such as air. This specifies the force due to friction on the oil droplets to be[5]

$$F = 6\pi r_d \eta v_t \tag{1}$$

where F is the force due to friction, $\mathrm{r_d}$ is the radius of the droplet, $\eta$ is the viscosity of the air, and $\mathrm{v_t}$ is the terminal velocity of the droplet.

At terminal velocity, the force due to friction and the force due to gravity are equal. Substituting the force due to gravity into (1) yields

$$mg = 6\pi r_d \eta v_t \tag{2}$$

where m is the mass of the droplet, and g is the acceleration due to gravity.

As the droplet is spherical, its mass is given by

$$m = \frac{4}{3}\pi r_d^3 \rho \tag{3}$$

where m is the mass of the droplet, $\mathrm{r_d}$ is the radius of the droplet, and $\rho$ is the density of the droplet.

The reticle on the microscope has two lines etched onto it at a fixed separation s. Thus $v_t$ can be calculated by measuring the time it takes an oil drop to traverse this separation, i.e. $v_t = \frac{s}{t_0}$ where s is the reticle separation and $t_0$ is the time it takes the falling drop to traverse that separation in the absence of an electric field. Substituting this into (2) and solving it for $r_d$ yields

$$m = \frac{9\sqrt{2}\pi \left(\frac{\eta s}{t_0 g}\right)^{\frac{3}{2}}}{\sqrt{\rho}} \tag{4}$$

where m is the mass of the droplet, $\eta$, s, $t_0$, g, and $\rho$ are as above.

When the droplet is rising at constant velocity under the influence of an electric field, the drag on the droplet, $F_d$, is given by Stokes' Law,

$$F_d = 6\pi \eta r_d v_u \tag{5}$$

where $v_u$ is the constant upward velocity of the droplet. For a given drop, we assume that its radius will remain constant. We also assume $\eta$ to be constant throughout the experiment. If we define $k = 6\pi\eta r_d$, this allows us to rewrite (2) as $mg = kv_t$, and (**??**) as $F_d = kv_u$. Substituting $v_u = \frac{s}{t_u}$, this ultimately allows us to rewrite $F_d$ as

$$F_d = mg\frac{t_0}{t_u} \tag{6}$$

where $t_u$ is the time it takes the rising droplet to traverse $s$. Since the drop is rising at constant velocity, the electric force must be equal in magnitude to the forces exerted by gravity and drag.

$$qE = mg + F_d \tag{7}$$

Where $q$ is the charge on the droplet, and $E$ is the intensity of the electric field. If we then substitute (6), (4), and $E = \frac{V}{d}$ into 7, we arrive at

$$q = \frac{9\sqrt{2}\pi d}{\sqrt{\rho g}V}\left(\eta\frac{s}{t_0}\right)^{\frac{3}{2}}\left(\frac{t_0}{t_u}+1\right) \tag{8}$$

where $d$ is the separation between the charged plates and V is the voltage across the two plates, as measured using a digital multimeter.

As our droplets are small, a correction factor must be applied to $\eta$. This correction factor is given by[6]

$$\eta_{eff} = \eta\left(\frac{1}{1+\frac{b}{Pr_d}}\right) \tag{9}$$

where $P$ is the air pressure in Pascals, and $b$ a constant[7]. Substituting this correction factor into 8 yields

$$q = \frac{9\sqrt{2}\pi d(t_0+t_u)}{t_0^{\frac{3}{2}}t_u V\sqrt{g\rho}}\left(\frac{s\eta}{1+\frac{2bg\rho}{-bg\rho+\sqrt{g\rho}\left(\frac{18P^2 s\eta}{t_0}+b^2 g\rho\right)}}\right)^{\frac{3}{2}} \tag{10}$$

which can be rewritten as

$$q = \frac{4}{3}\pi\rho g\left(\sqrt{(\frac{b}{2P})^2+\frac{9\eta s}{2g\rho t_0}}-\frac{b}{2P}\right)^2\frac{(t_u+t_0)sd}{Vt_0t_u} \tag{11}$$

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## IV. RESULTS AND DISCUSSION

A plot of our individual charge values shows evidence that the charge on each droplet is quantized.

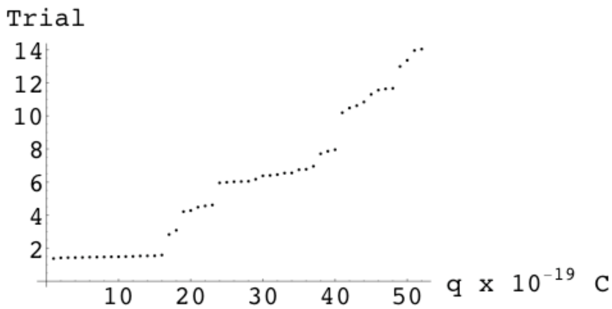


FIG. 3: Charge for each trial

With this quantization in mind, we divided our data into a number of bins, as shown in the table below.

TABLE I: Description of data bins

| Bin | Charge ($\times 10^{-19}$ C) |
|---|---|
| 1 | $1.47 \pm 0.05$ |
| 2 | $2.97 \pm 0.18$ |
| 3 | $4.43 \pm 0.17$ |
| 4 | $6.37 \pm 0.32$ |
| 5 | $7.84 \pm 0.12$ |
| 6 | $10.5 \pm 0.29$ |
| 7 | $11.5 \pm 0.17$ |
| 8 | $13.5 \pm 0.45$ |

We found the mean increase in charge to be $(1.72 \pm 0.19) \times 10^{-19}$ C. This is in agreement with the accepted value for the fundamental electric charge, $1.602 \times 10^{-19}$ C[8].

Our values for q are binned without regard to the logical drop[9] upon which they were measured. We had originally planned to calculate the average charge on each logical drop rather than binning the values as we did. Upon examination of our data, however, we found that for several logical drops the charge on the drop had changed significantly between trials.[10] This would have introduced a systematic error into our trials, so we decided against it.

[1] PASCO Scientific Model 300A Millikan Oil Drop Apparatus

[2] PASCO Scientific. *Instruction Manual and Experiment Guide for the PASCO Scientific Model AP-8210 Millikan Oil Drop Apparatus*. p. 19. Room temperature 23 degrees Centigrade.

[3] Air pressure for Charleston, SC on 21 April 2005. From Weather Underground. URL: http://www.wunderground.com/. Accessed 21 April 2005

[4] Mineral oil is sometimes referred to as "intestinal lubricant." Either nomenclature will suffice for the purposes of this experiment

[5] Melissinos, Adrian C. *Experiments in Modern Physics.* New York, Academic Press, 1966. p. 2)

[6] PASCO, *Op. Cit.* p. 2

[7] $8.2 \times 10^{-3} Pa \cdot m$. *Ibid.* p. 9

[8] Nave, C. Rod. "Electric Charge." From HyperPhysics. URL: http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html Accessed 2 May 2005

[9] we define a logical drop here to be a physical droplet whose charge has not been changed by means of the ionizing source.

[10] there are several plausible explanations for this, but they are not relevant here.